

Large N Expansion and Softly Broken Supersymmetry

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Abstract

We examine the supersymmetric non-linear $O(N)$ sigma model with a soft breaking term. In two dimensions, we found that the mass difference between supersymmetric partner fields vanishes accidentally. In three dimensions, the mass difference is observed but $O(N)$ symmetry is always broken also in the strong coupling region.

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1 Introduction

Supersymmetric field theories have many attractive features. For example, they may lead to the solution of the hierarchy problem or the non-renormalizability of the quantum gravity. While supersymmetry is theoretically attractive, it is not a manifest symmetry of nature. This necessitates the establishment of a realistic mechanism of supersymmetry breaking for these theories. In phenomenological models, we naively add soft breaking terms to the supersymmetric models and break supersymmetry at tree level.

In this letter we first re-examine the supersymmetric non-linear $O(N)$ sigma model in two and three dimensions. In two dimensions, both supersymmetry and $O(N)$ symmetry are not broken for any value of g . In three dimensions, however, we can find two phases. In the weak coupling phase, supersymmetry is not broken but $O(N)$ symmetry is broken. In the strong coupling phase, both supersymmetry and $O(N)$ symmetry are preserved. Next we examine the theory with a soft breaking term. In two dimensions, we found that the mass difference between supersymmetric partner fields accidentally vanishes but the supersymmetry is broken. In three dimensions, the mass difference is always observed and $O(N)$ symmetry is always broken also in the strong coupling region.

2 Large N expansion and softly broken supersymmetry

The supersymmetric non-linear sigma model is usually defined by the Lagrangian

$$L = \frac{1}{2} \int d^2\theta \Phi_j D^2 \Phi_j \quad (2.1)$$

with the non-linear constraint

$$\Phi_j \Phi_j = \frac{N}{g^2}. \quad (2.2)$$

where the sum of the flavor index j runs from 1 to N . The superfields Φ_j may be expanded out in components

$$\Phi_j = n_j + \bar{\theta} \psi_j + \frac{1}{2} \bar{\theta} \theta F_j \quad (2.3)$$

and the super covariant derivative is

$$D = \frac{\partial}{\partial \theta} - i \bar{\theta} \not{\partial}. \quad (2.4)$$

In order to express the constraint (2.2) as a δ function, we introduce a Lagrange multiplier superfield Σ .

$$\Sigma = \sigma + \bar{\theta}\xi + \frac{1}{2}\bar{\theta}\theta\lambda \quad (2.5)$$

We thus arrive at the manifestly supersymmetric action for the supersymmetric sigma model[1].

$$S = \int d^D x d^2\theta \left[\frac{1}{2}\Phi_j D^2\Phi_j + \frac{1}{2}\Sigma \left(\Phi_j\Phi_j - \frac{N}{g^2} \right) \right] \quad (2.6)$$

In the component form, the Lagrangian from (2.6) is

$$\begin{aligned} L = & -\frac{1}{2}n_j\partial^2 n_j + \frac{i}{2}\bar{\psi}_j \not{\partial}\psi_j + \frac{1}{2}F_j^2 - \sigma n_j F_j - \frac{1}{2}\lambda n_j^2 \\ & + \frac{1}{2}\sigma\bar{\psi}_j\psi_j + \bar{\xi}\psi_j n_j + \frac{N}{2g^2}\lambda \end{aligned} \quad (2.7)$$

We can see that λ, ξ , and σ are the respective Lagrange multiplier for the constraints:

$$\begin{aligned} n_j n_j &= \frac{N}{g^2} \\ n_j \psi_j &= 0 \\ n_j F_j &= \frac{1}{2}\bar{\psi}_j \psi_j \end{aligned} \quad (2.8)$$

The second and the third constraints of (2.8) are supersymmetric transformations of the first. We must not include kinetic terms for the field σ and ξ so as to keep these constraints manifest. We can derive gap equations from 1-loop effective potential or directly from eq.(2.8) by using the tadpole method[2]. These two approaches coincide to give the following equations.

(1) Scalar part

$$\begin{aligned} n_j n_j |_{m_n^2 = \langle \lambda \rangle + \langle \sigma \rangle^2} &= N \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \langle \lambda \rangle + \langle \sigma \rangle^2} \\ &= \frac{N}{g^2} \end{aligned} \quad (2.9)$$

(2) Fermionic part

$$\frac{1}{2}\bar{\psi}_j \psi_j |_{m_\psi = \langle \sigma \rangle} = n_j F_j \quad (2.10)$$

This relation includes auxiliary field F_j , to be eliminated by equation of motion. After substituting F_j by σn_j , we obtain:

$$n_j F_j = \sigma n_j n_j \quad (2.11)$$

If we impose the O(N) symmetric constraint $n^2 = \frac{N}{g^2}$, we have

$$\begin{aligned} \frac{N}{g^2} \sigma &= \frac{1}{2} \overline{\psi_j} \psi_j |_{m_\psi = \langle \sigma \rangle} \\ \frac{N}{g^2} &= \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \langle \sigma \rangle^2}. \end{aligned} \quad (2.12)$$

Let us examine these two equations in two and three dimensions.

For $D = 2$ we obtain from scalar part:

$$\begin{aligned} 1 &= \frac{g^2}{4\pi} \log \frac{\Lambda^2}{\langle \lambda \rangle + \langle \sigma \rangle^2} \\ m_n^2 &= \langle \lambda \rangle + \langle \sigma \rangle^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right) \end{aligned} \quad (2.13)$$

And from fermionic part:

$$\begin{aligned} m_\psi^2 &= \langle \sigma \rangle^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right). \end{aligned} \quad (2.14)$$

Substituting $\langle \sigma \rangle$ in the scalar constraint (2.13) with (2.14), we can find that $\langle \lambda \rangle$ must vanish. This means that ψ gains the same mass as n , and simultaneously the supersymmetric order parameter $\langle \lambda \rangle$ vanishes. We can say that the supersymmetry is not broken in two dimensions as is predicted by Witten[3].

For $D=3$, the situation is slightly different. We have a critical coupling constant g_{cr}^2 defined by:

$$1 = g_{cr}^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2}. \quad (2.15)$$

If we take $g^2 < g_{cr}^2$ something goes wrong with (2.9). It does not have any solution, so the constraint $\langle \vec{n}^2 \rangle = \frac{N}{g^2}$ cannot be satisfied. Of course, it is illusionary. We should also consider the possibility of spontaneous breaking of the O(N) symmetry. In above discussions, we have implicitly assumed that the vacuum expectation value of \vec{n} would vanish. Let us consider what may happen if \vec{n} itself gets non-zero vacuum expectation

value. Because of the $O(N)$ symmetry, the vacuum expectation value of $\vec{n} \equiv (n_1, n_2, \dots, n_N)$ may be written as

$$\langle \vec{n} \rangle = (0, 0, \dots, \sqrt{N}v/g). \quad (2.16)$$

So that the constraint equation (2.9) becomes

$$\begin{aligned} n_j n_j |_{m_n^2 = \langle \lambda \rangle + \langle \sigma \rangle^2} &= N \left(\frac{v^2}{g^2} + \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2 + \langle \lambda \rangle + \langle \sigma \rangle^2} \right) \\ &= \frac{N}{g^2}. \end{aligned} \quad (2.17)$$

Then we have another critical coupling constant g'_{cr} :

$$\frac{1 - v^2}{g_{cr}^2} = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \quad (2.18)$$

If g is smaller than g_{cr} , then v grows. As a result, the constraint equation has a solution in the weak coupling region ($g'_{cr} \leq g \leq g_{cr}$) in a sense that not eq.(2.9) but eq.(2.17) is satisfied by some $\langle \lambda \rangle + \langle \sigma \rangle^2$.

Then what will happen if we include the fermionic part? As far as $g \geq g_{cr}$, we have nothing to worry about. In the strong coupling region, both supersymmetry and the $O(N)$ symmetry are preserved as we have explained in two dimensional case. However, in the weak coupling region, the situation is changed. There is no non-trivial solution for the constraint (2.10) and there is no fermionic condensation that means no dynamical mass is generated for the fermion. It does not matter because we can set the supersymmetry breaking order parameter $\lambda = 0$ and then scalar field becomes massless as well. One may wonder why $\lambda = 0$ is favorable, but we can easily find that non-zero λ can be related to the positive vacuum energy if we also consider the effective kinetic term for the auxiliary superfield Σ .

So we can conclude:

(1) In two dimensions, both supersymmetry and the $O(N)$ symmetry are not broken. This means that λ and v remain zero for any value of g .

(2) In three dimensions, both supersymmetry and the $O(N)$ symmetry are not broken (i.e., λ and v remain zero) in the strong coupling region. The $O(N)$ symmetry can be broken in the weak coupling region, but supersymmetry is kept unbroken in both phases.

Now let us extend the above analysis to include a supersymmetry breaking mass term.

Here we consider:

$$L_{break} = m_s^2 n_j^2 \quad (2.19)$$

We can explicitly calculate the gap equation. For the scalar part:

$$\begin{aligned} n_j n_j |_{m_n^2 = \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} &= N \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} \\ &= \frac{N}{g^2} \end{aligned} \quad (2.20)$$

The fermionic part is unchanged by the breaking term. For $D = 2$ we can solve this equation explicitly.

$$\begin{aligned} 1 &= \frac{g^2}{4\pi} \log \frac{\Lambda^2}{\langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} \\ m_n^2 &= \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right) \end{aligned} \quad (2.21)$$

$\langle \sigma \rangle^2$ is determined by the fermionic part which is unchanged by the supersymmetry breaking term(2.19).

$$\begin{aligned} m_\psi^2 &= \langle \sigma \rangle^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right). \end{aligned} \quad (2.22)$$

These two equations suggest two consequences. One is that the supersymmetry breaking parameter λ gets non-zero value:

$$\langle \lambda \rangle + m_s^2 = 0 \quad (2.23)$$

So the supersymmetry is broken. The second is rather curious. As we can see from explicit calculations, dynamically generated masses are unchanged so the mass degeneracy is not removed. This happens because the auxiliary field λ has absorbed m_s so that the two masses balance.

So we conclude that, if we believe the large N expansion, the dynamical masses are unchanged while the supersymmetry breaking parameter develops non-zero value .

The crucial point of our observation lies in the fact that we can absorb the soft term by redefining a field. The simplest and trivial example is the ordinary $O(N)$ non-linear sigma model with an explicit mass term. This is written as:

$$L = -\frac{1}{2} n_j \partial^2 n_j - \frac{1}{2} \lambda (n_j^2 - \frac{N}{g^2}) - m^2 n_j^2 \quad (2.24)$$

Does the explicit mass term changes the dynamical mass? The answer is no. This can easily be verified by redefining λ as $\lambda' = \lambda + m^2$. Lagrangian is now:

$$L = -\frac{1}{2}n_j\partial^2n_j - \frac{1}{2}\lambda'(n_j^2 - \frac{N}{g^2}) - \frac{N}{2g^2}m^2 \quad (2.25)$$

We can find that the mass term is absorbed in λ and only a constant is left. Of course, this constant does not change the gap equation.

In three dimensions, however, it is not so simple. Many fields and their equations form complex relations and determine their values.

Let us see more detail. In three dimensions, we should slightly alter the above results. As is discussed above, this model has a weak coupling region where no dynamical mass is produced so no balancing effect between superpartner masses works in this region. Setting $\lambda = 0$, we find $m_n = m_s$ and $m_\psi = 0$ when g is small. This agrees with the naive expectation. What will happen if we go into the strong coupling region where the gap equation has non-trivial solution and the fermion becomes massive? If there is no soft term, $O(N)$ symmetry restoration occurs in this region. But when m_s is non-zero, v must develop non-zero value in order to compensate m_s and satisfy the constraint equation(2.17). In this case, we can set $\lambda = 0$ while v becomes non-zero.

To summarize, after adding a breaking term, some fields slide to compensate m_s but the mechanism is not trivial. Even in our simplest model, many complex relations determine their values.

3 Conclusion

We examined the supersymmetric non-linear $O(N)$ sigma model with a soft breaking term. In two dimensions, we found that the mass difference between supersymmetric partner fields vanishes accidentally but the supersymmetry is broken. In three dimensions, the mass difference is always observed but $O(N)$ symmetry is always broken even in the strong coupling region.

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